Theoretical Analysis on AC Resistance of Copper Clad Aluminum Wires

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Wireless power transfer system using inductive coupling through magnetic field requires low resistance coil at high frequency because its efficiency is significantly influenced by its quality factor. Copper clad aluminum (CCA) wire is an aluminum (Al) wire coated by a thin copper (Cu) layer. A CCA coil is not only light-weight and cost-effective, but also shows lower AC resistance than Cu one with the same dimension at high frequencies under certain circumstances. We formulated both the skin and proximity effects on CCA wires and analyzed numerically the AC resistance of the CCA-wound coils. The analysis has successfully explained the unusual phenomenon that CCA wires can suppress the AC resistance than Cu ones.

1. Introduction

A wireless power transfer (WPT) technology that transmits power using not electric wires but electromagnetic fields has recently attracted attention. This technology has been actively developed for applications not only in smartphones, tablets, and other small electronic devices but also to major appliances such as electric vehicles [1]. In a WPT system using inductive coupling through magnetic fields, the coil is an important part because its efficiency significantly depends on the quality factor $Q = \omega L/R$ of the coil, where ω, L and R are the angular frequency, inductance and resistance, respectively [2]. The coil resistance is required to be low at high frequencies as far as possible to improve the quality factor. However, the coil resistance increases as the frequency rises due to the skin and proximity effects both of which are caused by the eddy current inside of the wire generated by the flowing AC current. Increase of resistance not only lowers the power transmission efficiency but also increases the calorific values. For this reason, the AC resistance is required to be suppressed as much as possible.

A copper clad aluminum (CCA) wire is an aluminum (Al) wire surrounded by a thin copper (Cu) layer via strong metallic bond (Fig. 1). The CCA wire is mainly composed from Al that is cost-effective and lightweight than the commonly-used Cu. Furthermore, the connectivity and solderability of the CCA wire are similar to those of the Cu wire [3]. Since the conductivity of Al is lower than that of Cu, a CCA wire usually has larger diameter than the Cu wire; however, we found that the CCA coil shows lower AC resistance than the Cu coil even with the same shape under certain conditions, especially at high frequencies [4].

This paper analytically formulated the skin effect and proximity effect of the CCA wire to compare the AC resistance of the CCA coil with that of Cu coil based on numerical analysis results. We demonstrates that the loss of the CCA wire due to the proximity effect is smaller than that of the Cu wire thanks to the low conductivity of Al, and the CCA coil suppresses the increase in the AC resistance than that of the Cu coil. The numerical analysis results agree well with measurement results and simulate a phenomenon whereby AC resistance of the CCA coil is lower than that of Cu coil at high frequencies.

2. Formulation

2.1 AC Resistance Related to Skin Effect

Figure 2 shows the model of a circular wire uniformly distributed along z-direction with 2 layers, where the i-th layer has a conductivity of σ_i and a relative permeability of μ_i. Assuming a time factor of $e^{j\omega t}$, the z-component of electric field E_z at the i-th layer induced by a current I in z-direction satisfies

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} - j\omega\mu_i\mu_0\sigma_i E_z = 0, \cdots\cdots (1)$$

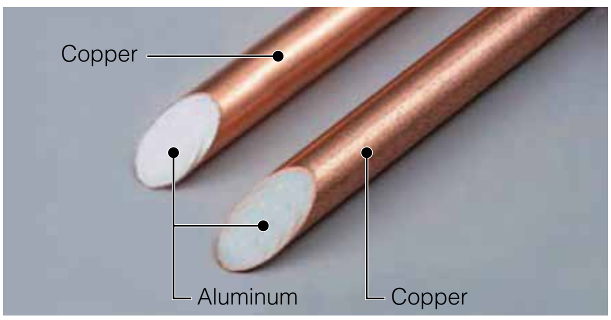

Fig. 1. Copper clad aluminum wires.

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Panel 1. Abbreviations, Acronyms, and Terms.

Quality factor
: It is defined as the ratio of the energy stored in a single cycle to that dissipated from the system in a resonance system. The quality factor of a coil is calculated by dividing the product of the inductance and angular frequency by the resistance.

Skin effect
: A phenomenon whereby AC current concentrates on the surface when it flows in conductive wire, thus reducing the effective area for the current to increase the AC resistance.

Proximity effect
: A phenomenon whereby the AC magnetic fields generated by the current flowing in proximity wires penetrate into wire each other, thus inducing the eddy current to generate loss and increase the AC resistance.

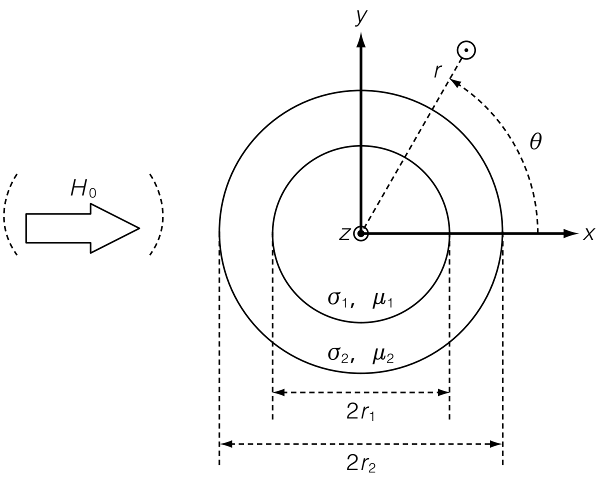

Fig. 2. Wire model for analysis.

which has a solution of

$$E_Z = \begin{cases} A_1 J_0(k_1 r) & (r \leqq r_1) \\ A_2 J_0(k_2 r) + B_2 Y_0(k_2 r) & (r_1 < r \leqq r_2) \end{cases}, \cdots\cdots (2)$$

where J_ν and Y_ν are Bessel and Neumann functions of ν-th order, respectively, $k_i^2 = -j\omega\sigma_i\mu_i\mu_0$, and A_i, B_i are constants yeilded by following boundary condision [5].

$$E_Z\big|_{r=r_1-} = E_Z\big|_{r=r_1+}, \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

$$\frac{1}{\mu_1}\frac{\partial E_Z}{\partial r}\bigg|_{r=r_1-} = \frac{1}{\mu_2}\frac{\partial E_Z}{\partial r}\bigg|_{r=r_1+}. \cdots\cdots\cdots (4)$$

Here a low-frequency approximation of $\omega\varepsilon_i \ll \sigma_i$ is generally available for metallic materials from DC to microwave frequency, where ε_i denotes the dielectic constant of the i-th layer. Equation (2) leads to the θ-component of magnetic field H_θ as

$$H_\theta = \begin{cases} -\dfrac{\sigma_1}{k_1} A_1 J_0'(k_1 r) & (r \leqq r_1) \\ -\dfrac{\sigma_2}{k_2}\left[A_2 J_0'(k_2 r) + B_2 Y_0'(k_2 r)\right] & (r_1 < r \leqq r_2) \end{cases}. \cdot\cdot(5)$$

According to Ampére's law, total current I flowing in the wire is given by enclosed integral of the magnetic field at the surface of the wire as

$$\begin{aligned} I &= \oint H_\theta\big|_{r=r_2} dl \\ &= \frac{2\pi\xi}{j\omega\mu_2\mu_0}\left[A_2 J_0'(\xi) + B_2 Y_0'(\xi)\right], \cdots\cdots\cdots (6) \end{aligned}$$

where $\xi = k_2 r_2$. Then all the constants are related to I.

Loss of the wire with a length of l is obtained by power flow passing into the wire through the surface as

$$\begin{aligned} \bar{P}_S &= -\frac{1}{2}\oint E \times H^* \cdot dS \\ &= \frac{j\omega\mu_2\mu_0 l|I|^2}{4\pi\xi} \cdot \frac{A_2 J_0(\xi) + B_2 Y_0(\xi)}{A_2 J_0'(\xi) + B_2 Y_0'(\xi)}. \cdots (7) \end{aligned}$$

On the other hand, the loss of the wire with resistance of R and inducntance of L is given as

$$\bar{P}_S = \frac{1}{2}(R + j\omega L) l |I|^2. \cdots\cdots\cdots\cdots\cdots (8)$$

By equating of (7) with (8), the resistance per unit length is given as

$$R_S = \Re\left[\frac{j\omega\mu_2\mu_0}{2\pi\xi} \cdot \frac{A_2 J_0(\xi) + B_2 Y_0(\xi)}{A_2 J_0'(\xi) + B_2 Y_0'(\xi)}\right], \cdots (9)$$

where $\Re$ denotes the real part of complex number.

2.2 Loss Caused by Proximity Effect

Assuming that a homogenous AC magnetic field with intensity of H_0 is applied to the wire along x-direction as shown in Fig.2. The z-component of a magnetic potential A_z satisfies

$$\frac{\partial^2 A_Z}{\partial r^2} + \frac{1}{r}\frac{\partial A_Z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A_Z}{\partial \theta^2} + k_i^2 A_Z = 0, \cdots (10)$$

which has a solution of

$$A_Z = \sin\theta \times \begin{cases} C_1 J_1(k_1 r) & (r \leqq r_1) \\ C_2 J_1(k_2 r) + D_2 Y_1(k_2 r) & (r_1 < r \leqq r_2), \cdots (11) \\ C_3 r + D_3 r^{-1} & (r_2 < r) \end{cases}$$

where C_n and D_n are constants yielded by the following boundary condition.

$$\mu_i A_Z \big|_{r=r_i-} = \mu_{i+1} A_Z \big|_{r=r_i+} \ (i = 1,\ 2), \cdots\cdots (12)$$

$$\frac{\partial A_Z}{\partial r}\bigg|_{r=r_i-} = \frac{\partial A_Z}{\partial r}\bigg|_{r=r_i+} \quad (i=1,\ 2). \cdots\cdots (13)$$

In addition, $A_z = H_0 \sin\theta$ at $r \to \infty$. This gives

$$C_3 = H_0 . \cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

Then all the constants are related to H_0.

Loss due to eddy current in the wire with length of l is calculated by the power flow passing through the surface of the wire as

$$\bar{P}_p = -\frac{1}{2}\oint E \times H^* \cdot dS$$
$$= -\frac{2\pi l |\xi|^2 |H_0|^2}{\sigma_2} \cdot \frac{\xi X Y^*}{|Z|^2}, \cdots\cdots (15)$$

where

$$X = C_2 J_1(\xi) + D_2 Y_1(\xi),$$
$$Y = C_2 J_1'(\xi) + D_2 Y_1'(\xi), \cdots\cdots (16)$$
$$Z = (\mu_2 - 1) X + \xi [C_2 J_0(\xi) + D_2 Y_0(\xi)].$$

Real part of $\bar{P}_p$ gives the eddy current loss.

2.3 AC Resistance of Coils

For a coil wound by wire, magnetic field is generated by the current flowing in the wire. Therefore,the magnitude of the field is proportional to the magnitude of the current, i.e.

$$|H_0| = \alpha |I|, \cdots\cdots\cdots\cdots\cdots\cdots (17)$$

where α is a shape factor which depends on the structure of the coil. By introducing this factor, the total AC resitance of the wire is given by

$$R_{ac} = R_S + \alpha^2 D_p, \cdots\cdots\cdots\cdots\cdots (18)$$

where D_P is associated with the loss caused by the proximity effect per unit length and given by

$$D_p = -\frac{4\pi|\xi|^2}{\sigma_2} \cdot \Re\left(\xi \frac{XY^*}{|Z|^2}\right). \cdots\cdots (19)$$

3. Numerical Results

3.1 Skin Effect

Figure 3 shows R_s of the Cu and 5% CCA (the area ratio of Cu to the cross section is 5%) wires with 0.4 mm diameter. The conductivity of Cu and that of Al are 5.8×10^7 and 3.3×10^7 S/m, respectively. Although R_s of the CCA wire gradually approaches that of the Cu wire at high frequencies, it is higher than that of the Cu wire at all frequencies of up to 100 MHz.

Figure 4 indicates the distribution of the current density inside the same wires at 500 kHz normalized by uniform current of the Cu wire. Because of the difference in conductivity, the current density inside of the CCA wire discontinuously increases at the interface between the Al and Cu layers of r = 0.195 mm.

Figure 5 shows the loss distribution inside the wires when a current of 1 A and 500 kHz flows. Since the Cu

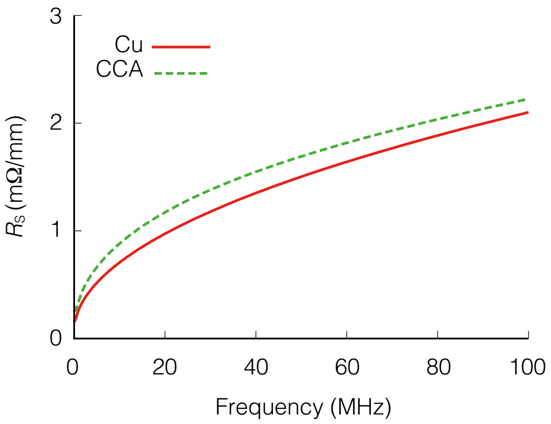

Fig. 3. Calculated *Rs* in CCA and Cu wires.

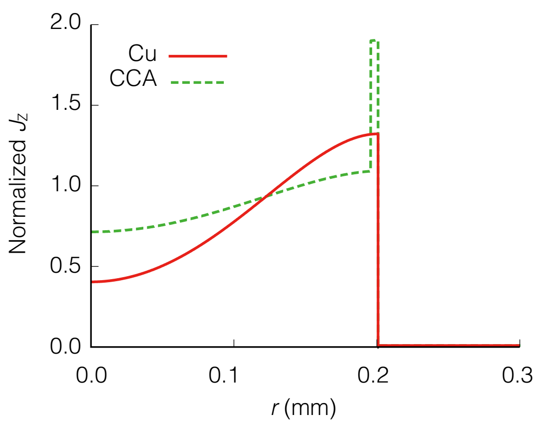

Fig. 4. Distribution of current density at 500 kHz.

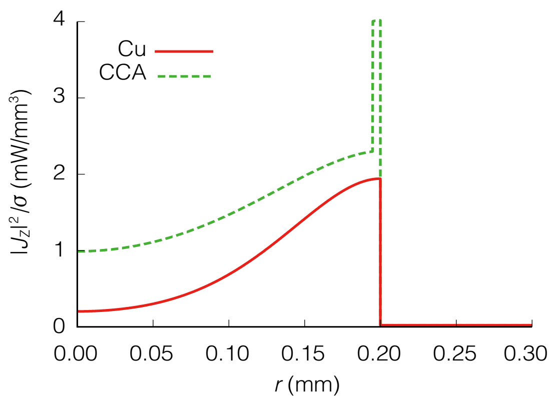

Fig. 5. Distribution of loss density at 500 kHz.

layer of the CCA wire has a higher current density, the current density of the Al layer is lower than that of the uniform Al wire; however the loss density of the CCA wire is higher than that of the Cu wire because of the difference in conductivity. For this reason, *Rs* of the CCA wire is not lower than that of the Cu wire at all frequencies.

3.2 Proximity Effect

Figure 6 illustrates the calculated D_p due to the proximity effect of the same wires. The loss of the CCA wire is lower than that of the Cu wire at 420 kHz or less and higher at 420 kHz or more.

The reason why D_p of CCA wire is lower than that of Cu wire is explained as follows. Figure 7 shows the distribution of the magnetic field around the wires on the *y*-axis when an external AC magnetic field of 1 A/mm and 100 kHz is applied from the *x*-axis direction, and Fig. 8 shows distribution of the corresponding eddy current loss. As shown in Fig. 7, the magnetic field concentrates on the surface of the wires at high frequencies, and the time variation of the magnetic field in the CCA wire is smaller than Cu wire. Since the eddy current is proportional to the degree of variation in magnetic fields, the eddy current loss density of the CCA wire is lower than that of the Cu wire; however, when the frequency rises further, most of the magnetic fields concentrate on the surface of the conductor and the distribution of eddy current density of the CCA wire becomes similar to that of the Cu wire. This additionally increases the eddy current loss of the low conductivity CCA wire. Figure 9 illustrates the distribution of the eddy current loss density when applying an AC magnetic field of 1 A/mm and 1 MHz. It identifies that the eddy current density of the CCA wire is higher than that of the Cu wire.

Let us further qualitatively examine the phenomenon whereby the proximity effect loss of the CCA wire is lower than that of the Cu wire at low frequencies and higher at high frequencies. For simplification, the wire is assumed to be round wire with a radius and be made of only single kind of material. The loss of this conductor caused by the proximity effect is expressed by the following equation to which Eq. (19) is simplified [6]

$$D_p = 4\pi\omega\mu a^2 f_{\mathrm{prox}}(\zeta) \quad \cdots\cdots\cdots\cdots\cdots (20)$$

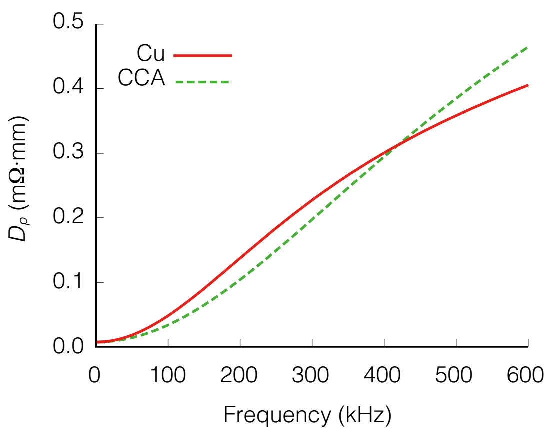

Fig. 6. Calculated D_p in CCA and Cu wires.

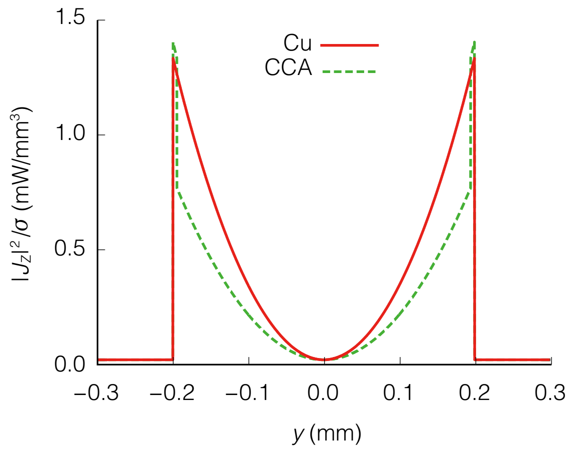

Fig. 8. Distribution of eddy-current loss density at 100 kHz.

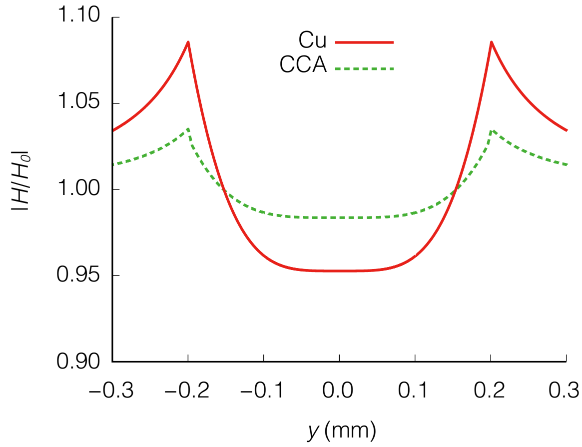

Fig. 7. Distribution of magnetic field at 100 kHz.

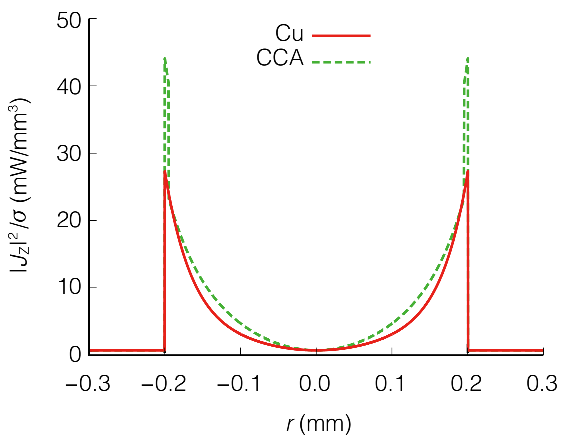

Fig. 9. Distribution of eddy-current loss density at 1 MHz.

where

$$f_{\text{prox}}(\zeta) = \frac{1}{\zeta}\,\frac{\text{ber}\zeta \cdot \text{ber}'\zeta + \text{bei}\zeta \cdot \text{bei}'\zeta}{\text{ber}^2\zeta + \text{bei}^2\zeta}, \quad \cdots\cdots\cdots (21)$$

$$\zeta = \frac{\sqrt{2}\,a}{\delta} = \sqrt{\omega\mu\sigma}\,a, \quad \cdots\cdots\cdots (22)$$

and μ and σ are relative permeability and conductivity, respectively, and ber, bei are Kelvin functions. In addition, $\delta = \sqrt{2/(\omega\mu\sigma)}$ is the skin depth and the variable ζ is a normalized radius. Figure 10 and 11 illustrate the magnetic field distributions around the conductor when the magnetic field is applied to the conductor. Both figures assume that $a/\delta = 1$ or 10. The larger ζ is, the stronger the deviation of the magnetic field. In addition, when ζ is constant, the magnetic field distribution normalized by the wire radius remains unchanged.

The function f_{prox} is common to all wires with a round cross section because it is a function of ζ. In addition, since this function isolates ζ from D_p, it gives the σ dependence on D_p where a, ω, and μ are fixed. As shown in Fig. 12, f_{prox} is an increasing function within a range of $\zeta < 2.5$ and is a decreasing function within a range of $\zeta > 2.5$. In other words, with regard to the boundary of $\zeta = 2.5$, the higher the conductivity, the larger the loss is when a, ω, and μ are small. When a, ω, and μ are large, the lower the conductivity, the smaller loss is. This explains the behavior of the CCA wire's loss due to the proximity effect.

3.3 AC Resistance for Coils

Figure 13 shows the measured and calculated AC resistances of the coils wound by cable stranded with

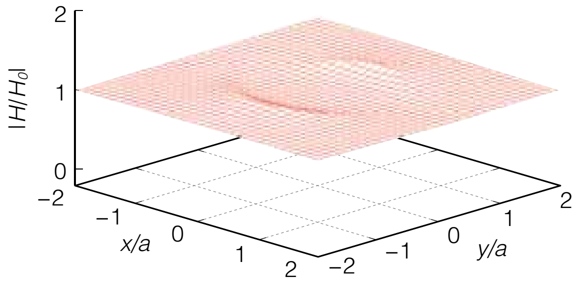

Fig. 10. Magnetic field distribution for *a* / *δ* = 1 when external field is applied.

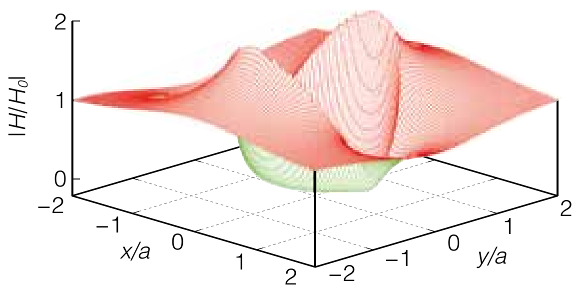

Fig. 11. Magnetic field distribution for *a* / *δ* = 10 when external field is applied.

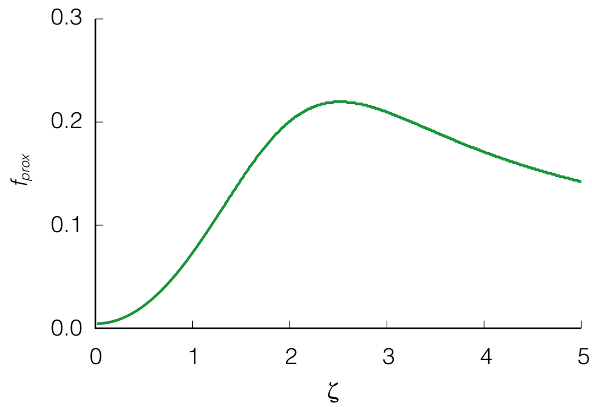

Fig. 12. Function of proximity effect.

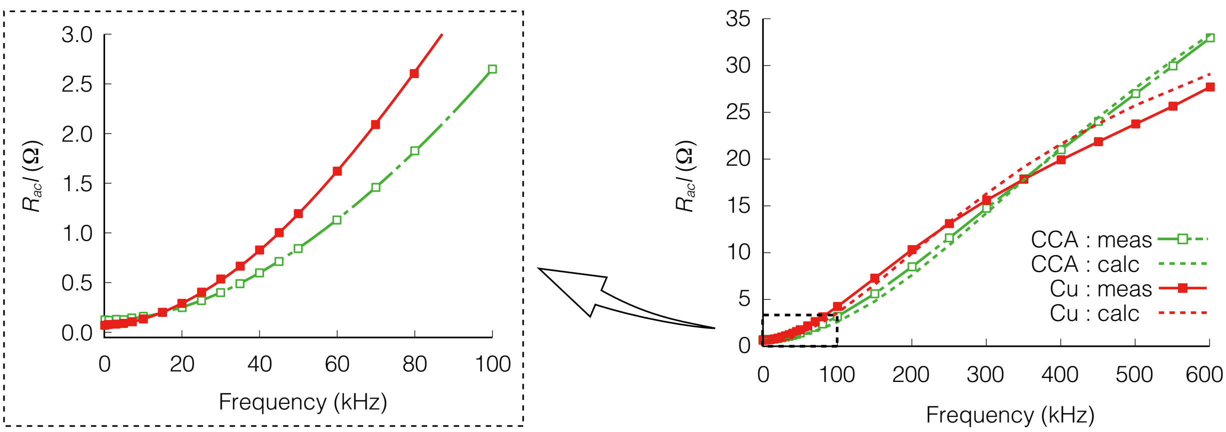

Fig. 13. Comparison of AC resistance.

Fig. 14. Coil wound by CCA wires.

14 wires with a diameter of 0.4 mm for 8 layers of 10 turns on a bobbin with a diameter of 20 mm, as shown in Fig. 14. The length of each conductor is 7.2 m, which is enough short with respect to the wavelength to treated as a lumped constant circuit. The shape factor α is calculated as 11.8 mm^{-1} by fitting measured and calculated values using the least-squares method. The measured values agreed well with the calculated values; we simulates a phenomenon whereby the AC resistance of the CCA coil is lower than that of the Cu coil at a frequency from 15 to 350 kHz. At 60 kHz, the resistance of the CCA wire is 69% of that of the Cu wire; the coil on which the conductors with the same diameter were wound achieved 30% or more reduction in resistance.

4. Conclusion

In this paper, we numerically analyzed the AC resistance of the wire with a round cross section and 2-layers due to the skin and proximity effects. In addition, we theoretically determined a phenomenon whereby the eddy current loss of the CCA wire is smaller than that of the Cu wire at lower frequencies. We performed measurement and numerical calculation to show a phenomenon whereby the AC resistance of the coil of the CCA wire is lower than that of the coil of the Cu wire. The CCA wire is particularly useful for conductors for wireless power transfer with which mobile units such as electric vehicles are equipped because it is light in weight and can be used as a low-loss conductor at high frequencies.

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